Enrollment No:-__________ Exam Seat No:-_______

C.U.SHAH UNIVERSITY

Summer-2015

Subject Code4SC04MTC1 **Subject Name: : Differential and Integral Calculus**

Course Name: B.Sc. Date: 19/5/2015

Semester: IV Marks: 70

Time: 10:30 TO 01:30

Instructions:

1) Attempt all Questions in same answer book/Supplementary.
2) Use of Programmable calculator & any other electronic instrument prohibited.
3) Instructions written on main answer book are strictly to be obeyed.
4) Draw neat diagrams & figures (if necessary) at right places.
5) Assume suitable & perfect data if needed.

SECTION-I

Q-1 a) If $\emptyset(x,y,z) = x^3y + xy^3 + xyz$ find $\nabla\emptyset$. (02)

b) Check whether the vector field $\vec{F} = (x^2 + xy^2)i + (y^2 + x^2y)j$ is irrotational. (02)

c) Evaluate $\int_0^2 \int_0^1 (x^2 + 2y)\, dx\, dy$ (02)

d) What does $\iint_R dxdy$ geometrically represent? (01)

Q-2 a) For a function $\bar{f}$ whose second order derivative exists then prove that $curl\ \bar{f}$ is solenoidal. (05)

b) If $u = x + y + z, v = x^2 + y^2 + z^2, w = yz + xz + xy$ then prove that $grad\ u, grad\ v$ and $grad\ w$ are coplanar. (05)

c) If $\bar{f} = grad\ (x^3 + y^3 + z^3 - 3xyz)$ then find $grad\ (div\ \bar{f})$. (04)

OR

Q-2 a) For a solenoidal vector field $\vec{H}$ show that $curl\ curl\ curl\ curl\ \vec{H} = \nabla^4\vec{H}$. (05)

b) Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dx\, dy$ by changing the order of integration. (05)

c) Find unit normal vector to the surface $\emptyset = x^2y + y^2z + z^2$ at the point $(1,1,1)$. (04)

Q-3 a) Evaluate $\iint_R y\, dx\, dy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. (05)

b) Find the value of $\iint_R (1 - x^2 - y^2)\, dx\, dy$ where R is circle $x^2 + y^2 \le 1$. (05)

c) Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} r^2\, dr\, d\theta$. (04)

OR

Q-3 a) Using multiple integral, prove that volume of sphere having radius a is $\frac{4}{3}\pi a^3$. (05)

b) Applying the transformation $u = \frac{2x-y}{2}$, $v = \frac{y}{2}$ and evaluate (05)

$$\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x-y}{2}\, dx\, dy.$$

c) Evaluate: $\int_0^1 dx \int_0^2 dy \int_1^2 x^2yz\, dz$ (04)

Section – II

Q-4 a) Verify $u = e^x \cos y$ is a solution of partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$ (02)

b) Find tangents at origin and give the nature of double point $(0,0)$. (02)

c) Form a partial differential equation from the equation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$. (02)

d) State Stoke's theorem. (01)

Q-5 a) Verify Green's theorem for $\int_c (x+y)dx + 2xy\, dy$, where c is the rectangle in $XY-$ plane bounded by $x = 0, y = 0, x = a$ and $y = b$. (07)

b) Evaluate $\iint_s \vec{F} \cdot \hat{n}\, ds$, where $\vec{F} = yi + zj$ and s is a part of $2x + 2y + z = 2$. (04)

c) Evaluate $\int_{(2,1)}^{(1,2)} y\, dx$over the line segment joining the points $(2,1)$ to $(1,2)$. (03)

OR

Q-5 a) Using Green's theorem evaluate $\int_c (x+y^2)dx + (x^2-y)dy$, where c is curve between $y^2 = x$ and $y = x$. (06)

b) If $\vec{F} = 3xy\, i - y^2 j$, evaluate $\int_c \vec{F} \cdot d\bar{r}$, where c is the curve in $XY-$ plane $y = 2x^2$ from $(0,0)$ to $(1,2)$. (04)

c) Verify stoke's theorem for $\bar{F} = (x^2 - y^2)i + 2xy\, j$ in the rectangular region $x = 0, y = 0, x = a, y = b$. (04)

Q-6 a) If ρ be the radius of curvature for the polar curve $r = f(\theta)$ then prove that $\rho = \frac{(r^2+r_1^2)^{\frac{3}{2}}}{r^2 - r\, r_2 + 2\, r_1^2}$. (06)

b) Find multiple points on the curve $x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$. Also determine it's type. (04)

c) Solve: $x\, p + y\, q = 3z$. (04)

OR

Q-6 a) Prove that radius of curvature for the curve $y = f(x)$ is $\frac{(1+y_1^2)^{\frac{3}{2}}}{y_2}$. (06)

b) Find multiple points on the curve $(y-6) = x^2(x-2)^3 - 9$. Also determine it's type. (04)

c) Solve: $\frac{y^2 z}{x}\, p + xz\, q = y^2$ (04)